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P. Castiglione, M. Falcioni, A. Lesne, A. Vulpiani: Chaos and Coarse Graining in Statistical Mechanics

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This is an excellent book written in highly polished fashion by four experts in the fields of dynamical systems and statistical mechanics. The discussions are clear and forthright, and no attempt is made to obfuscate or to hide things under a rug. Because of this, the title is somewhat misleading.

The book is divided into three parts: deterministic chaos and complexity (Chaps. 1–3), foundation of equilibrium and nonequilibrium statistical mechanics (Chaps. 4–6); and effective equations, multiscale and renormalization group (Chaps. 7 and 8). Two general types of systems are discussed; those with small numbers of particles and degrees of freedom and those with essentially an infinite number of particles and degrees of freedom.

The first part of the book is an excellent presentation of deterministic chaos and complexity theory in finite systems. This part of the book is one of the clearest and most insightful discussions I have run across and could well serve as text for a course in dynamical systems. Chapter 3, in particular, introduces the concepts of finite size Lyapunov exponents and ε entropy. These concepts are used to justify the fact that systems are measured for finite times and ergodic and Poincaré recurrences are not directly applicable. The distinction between microscopic and macroscopic chaos is introduced and hydrodynamic Lyapunov exponents are described. Self-organized criticality is mentioned in passing. The important point is made that with finite data sets, it is extremely difficult to distinguish between deterministic and stochastic systems.

The second part of the book on the foundation of statistical mechanics is the most interesting and instructive for me. It is devoted to a discussion of the connection between dynamical behavior involving ergodicity and chaos and statistical mechanics. The approaches of Boltzmann and Gibbs are discussed, and the common ground is emphasized. The point is made that while the ergodicity of a system has not been measured and is not a useful concept, what is essential is the ergodicity of reduced functions in the system. The authors' conclusions are that ergodicity and chaos cannot be used to justify either equilibrium or

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nonequilibrium statistical mechanics. It is because of this that I think the title of the book is somewhat misleading. It appears that the most important property is the size of the system.

Chapter 5 is entitled: On the Origin of Irreversibility. There are interesting discussions in this chapter, but the presentations and arguments are not as clear-cut as in the previous chapter. The explanations of irreversibility as due to changes in the fundamental dynamical equations or chaos are correctly disregarded. The basic clue is mentioned but not developed. Namely, the laws of thermodynamics are not exact on a molecular scale. The clues are apparent in the results of two experiments. The spin-echo experiment of Hahn and the Gibbs *gedanken* ink blot experiment. In both cases, the system starts out of equilibrium and appears to go to an equilibrium state macroscopically. However, the memory of the initial state remains in the apparent equilibrium state and the initial state can be recovered in practice and in principle. As stated in the texts, "Once the equilibrium values have been attained, a change is almost impossible since the microstates of the other classes represent an irrelevant fraction of the total." Thus, if the system starts off with "good" initial conditions, irreversibility will seem to result.

In Chap. 6, the arguments of Chap. 5 are further developed. Here again, it appears that chaos is not crucial for nonequilibrium statistical mechanics. There does appear to be a weak link between the Kolmagorov Sinai entropy and the Gibbs entropy. While the Gibbs and Boltzmann expressions for the entropy yield compatible results in equilibrium, they do not out of equilibria. The Boltzmann expression involves the time dependent singlet distribution function. If the Gibbs formulation is written in terms of the time dependent N particle distribution function, the result is patently incorrect. A good approximation out of equilibrium involves the substitution of the local equilibria distribution function in place of the exact time dependent distribution function. This is the analog of the Boltzmann form. There is a discussion of linear response theory and van Kampen's objections! His objection is due to the fact that the N particle functions do not decay to equilibrium. However, the reduced distribution functions do approach the equilibrium form.

Chapter 7 discusses coarse graining equations in complex systems. It continues the arguments in Chaps. 5 and 6.

Finally, Chap. 8 discusses renormalization-group approaches. The connection between the renormalization group and coarse graining is emphasized.

This is an excellent book for experts in the field and would provide a provocative textbook for graduate students. I couldn't find any typos or misstatements in the book.